UNIVERSITÄT SIEGEN
Fakultät IV
Department Mathematik

Stochastic dynamic optimization for energy market problems

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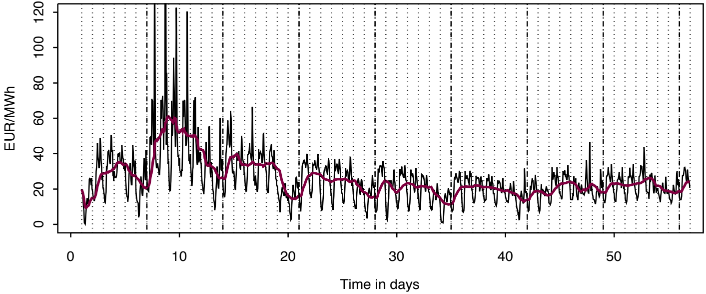
Electricity Prices

Hourly electricity prices at the European Energy Exchange:



Electricity Prices

Properties of hourly electricity prices S_t:

- daily, weekly and yearly seasonalities,
- extreme price spikes,
- strong mean reversion,
- state dependent volatility,
- long-term non-stationarity.

$\rightarrow$ complicated stochastic process.

Electricity Prices

SMaPS-Modell of Burger et al. (2004):
(spot market price simulation model)

$$\ln(S_t) = f(t, L_t/v_t) + X_t + Y_t, \quad t = 0, 1, 2, \ldots.$$

mit

- $(L_t)_{t\geq 0}$: load process
- $f(t,\cdot),\ t \geq 0$: (logarithmic) price-load curves
- $(v_t)_{t\geq 0}$: expected availability of power plants
- $(X_t)_{t\geq 0}$: (stationary) short term process
  (calibrated to spot market data)
- $(Y_t)_{t\geq 0}$: (non-stationary) long term process
  (calibrated to futures data)

$(L_t), (X_t), (Y_t)$ independent.

Electricity Prices

price-load curves $f(t, \cdot)$:
- cubic splines fitted to data

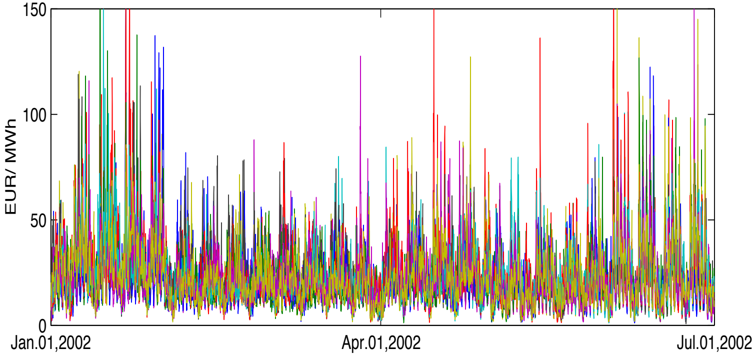
short term process X_t:
SARIMA$(2, 0, 1) \times (1, 0, 1)_{24}$ time series model

load process $L_t = \ell_t + L'_t$:
ℓ_t: deterministic load forecast
(describes the main seasonalities!)
L'_t: SARIMA$(1, 0, 1) \times (1, 0, 1)_{24}$ time series model

long term process Y_t:
- random walk mit drift

Electricity Prices

Simulation model:



20 simulations of price paths

Dynamic Optimization Problems

Stochastic dynamic optimization problems in energy markets

- valuation of gas storage facilities

- valuation of swing options

...

Example Swing Option

fixed energy amount: A
capacity limit: C
delivery spread over time points $t_1, \dots, t_N$.
strike price K
Optimization problem:

$$
\begin{aligned}
V_{Swing} \quad &= \quad \max_{\phi} \left\{ \sum_{i=1}^{N} E^* \left[\phi(t_i) \left(S_{t_i} - K \right) \right] \right\} \qquad (1) \\
\text{s.t.} \quad & \sum_{i=1}^{N} \phi(t_i) = A, \quad 0 \le \phi(t_i) \le C \;\; (i = 1, \dots, N), \\
& \phi(t_i) \quad \mathcal{F}_{t_i} - \text{adapted} \;\; (i = 1, \dots, N),
\end{aligned}
$$

with $\phi(t)$ capacity at time t.
Optimal solution??

Example Swing Option

Assume that $S_1, S_2, \ldots$ are independent.
$\rightarrow$ Markov decision process, known as
"stochastic sequential assignment problem":
Let $V_t(e,p)$ optimal expected reward from t on, if energy amount e is still available and current price is p.
Bellman equations:

$$V_t(e,p) = \max_{0 \le \phi(t) \le C} \{\phi(t)(p-K) + EV_{t+1}(e-\phi(t), S_{t+1})\}$$

Optimal policy is **Swing**-strategy of threshold type:

$$\phi(t) = \begin{cases} C & , \text{ if } p \geq k_t(e), \\ 0 & , \text{ else,} \end{cases}$$

for some k_t.

Example Swing Option

Asymptotically optimal for large N:

$$\phi(t) = \begin{cases} C & , \text{ if } p \geq F_S^{-1}(1-\alpha), \\ 0 & , \text{ else,} \end{cases}$$

where $\alpha = A/CN$.

Is this still true for more realistic stochastic processes (S_t)?

Example Swing Option

Assume that $S_1, S_2, ...$ is Markovian process with $S_{t+1} = g(S_t, Y_{t+1})$.
Bellman equation:

$$V_t(e, s_t) = \max_{0 \le \phi(t) \le C} \{\phi(t)(s_t - K) + E[V_{t+1}(e - \phi(t), S_{t+1}) | S_t = s_t]\}$$

Optimal policy is still **bang-bang**-strategy, i.e. $\phi(t) \in \{0, C\}$.
This holds under very general assumptions.

-> reduction to multiple stopping problem with n stopping times.

$$V_t(n, z_t) = \max\{z_t + E[V_{t+1}(n-1, Z_{t+1}) | Z_t = z_t], \\ E[V_{t+1}(n, Z_{t+1}) | Z_t = z_t]\}$$

Example Swing Option

Optimal policy is not necessarily threshold strategy.
Example: let $S_1 \sim U(2,4)$, $S_2 = 2S_1 - 3 \sim U(1,5)$, $n = 1$ (optimal stopping problem).

Obvious solution: stop in $t = 1$, if $S_1 \leq 3$!!

Optimal policy is of threshold type under the assumption that

$$\|g(\cdot, y)\|_L \leq 1 \text{ for all } y.$$

Example Swing Option

In general no explicit solution in case of complicated stochastic process for spot price S_t.
-> need for good approximations, upper and lower bounds.

Any admissible policy delivers a lower bound. A very simple lower bound used in practice is given by the *ex ante* strategy looking only at the current futures prices.

A very simple upper bound is given *ex post* by the so called *prophet value*.

Example Swing Option

Better bounds:

Good lower bounds by *approximate dynamic programming* with simulation algorithms like *least square Monte Carlo*.

Good upper bounds by duality theory developped by Rogers (2002) and Meinshausen and Hambly (2004).

Explanation with example of optimal stopping ($n = 1$).

Example Swing Option

Given any stochastic process $(Z_t, t = 0, 1, \ldots, T)$ the following result of Rogers (2002) holds.

Theorem

$$\sup_{\tau} E(Z_\tau) = \inf_{M \in H_0} E[\sup_t (Z_t - M_t)]$$

where H_0 is the set of all martingales with $M_0 = 0$.

The infimum is attained for M^* the martingale from the Doob-Meyer-decomposition of the Snell envelope

$$M_t^* - M_{t-1}^* = V_t(Z_t) - E[V_t(Z_t)|Z_{t-1}].$$

-> any good policy (approximation $\tilde{V}$ of the value function) delivers also a good upper bound via

$$M_t - M_{t-1} = \tilde{V}_t(Z_t) - E[\tilde{V}_t(Z_t)|Z_{t-1}].$$

Example Swing Option

In case of an ergodic Markov chain (S_t) a good policy for large N is given (under a few technical assumptions) by the assymptotically optimal strategy

$$\phi(t) = \begin{cases} C & \text{, if } p \geq \pi^{-1}(1-\alpha), \\ 0 & \text{, else,} \end{cases}$$

where $\alpha = A/CN$ and π is the stationary distribution of the Markov chain.

Approximate dynamic programming

An approximation $\tilde{V}$ of the value function can also be obtained via *least square Monte Carlo*, suggested by Longstaff and Schwartz (2001).

Example of a put option with $T = 3, K = 1.05, r = 0.02$.

Given: 10 simulated paths of a stochastic process S_t.

Goal: approximation of the value of the option and good exercising strategy.

We approximate the continuation value $E[\tilde{V}_t(S_t)|S_{t-1}]$ via polynomials of 2. degree (basis functions $1, x, x^2$).

Approximate dynamic programming

path	$t = 0$	$t = 1$	$t = 2$	$t = 3$
1	1.00	1.06	1.10	1.27
2	1.00	1.18	1.04	0.99
3	1.00	0.91	0.95	0.94
4	1.00	0.80	0.75	0.90
5	1.00	1.10	1.30	1.38
6	1.00	1.01	1.03	1.15
7	1.00	1.11	1.56	1.44
8	1.00	0.89	0.84	0.90
9	1.00	0.95	0.95	0.96
10	1.00	1.20	1.05	1.07

Table: Simulated paths of the stochastic process.

Approximate dynamic programming

path	$t = 1$	$t = 2$	$t = 3$
1	-	-	0.00
2	-	-	0.06
3	-	-	0.11
4	-	-	0.15
5	-	-	0.00
6	-	-	0.00
7	-	-	0.00
8	-	-	0.15
9	-	-	0.09
10	-	-	0.00

Table: Cash Flows Z_3 if exercising only at the end.

Ansatz: $E[Z_3|S_2] \approx a + bS_2 + cS_2^2$

Approximate dynamic programming

Least squares method delivers the regression:

$$E(Z_3|S_2) \approx -0.9415 + 2.8064\, S_2 - 1.8033\, S_2^2.$$

Comparing current value $(K - S_2)_+$ with the estimated continuation values gives a (sub)-optimal strategy for $t = 2$.

Approximate dynamic programming

Path	$(K - S_2)_+$	est. cont. value
1	-	-
2	0.01	0.0266
3	0.10	0.0970
4	0.30	0.1489
5	-	-
6	0.02	0.0359
7	-	-
8	0.21	0.1434
9	0.10	0.0970
10	-	-

Table: Value of exercising the option in $t = 2$ and estimated continuation value.

For path 2 and 6 wait, otherwise exercise in $t = 2$.

Approximate dynamic programming

path	$t = 1$	$t = 2$	$t = 3$
1	-	0.00	0.00
2	-	0.00	0.06
3	-	0.10	0.00
4	-	0.30	0.00
5	-	0.00	0.00
6	-	0.00	0.00
7	-	0.00	0.00
8	-	0.21	0.00
9	-	0.10	0.00
10	-	0.00	0.00

Table: Cash Flows of exercising in $t = 2$ or $t = 3$.

Approximate dynamic programming

Now regression of the discounted cash flow Z_2 in relation to the current price S_1. Least squares method gives:

$$E(Z_2|S_1) \approx 2.0789 - 2.9407\, S_1 + 0.8919\, S_1^2.$$

Compare the current value $(K - S_1)_+$ with the estimated continuation value $E(Z_2|S_1)$ to get a (sub)-optimal strategy in $t = 1$.

Approximate dynamic programming

path	exercising	est. cont. value
1	-	-
2	-	-
3	0.14	0.1414
4	0.25	0.2972
5	-	-
6	0.04	0.0186
7	-	-
8	0.16	0.1682
9	0.10	0.0902
10	-	-

Table: value of exercising in $t = 1$ and estimated continuation value.

For paths 6 and 9 the option should be exercised in $t = 1$.

Approximate dynamic programming

path	$t = 1$	$t = 2$	$t = 3$
1	0.00	0.00	0.00
2	0.00	0.00	0.06
3	0.00	0.10	0.00
4	0.00	0.30	0.00
5	0.00	0.00	0.00
6	0.04	0.00	0.00
7	0.00	0.00	0.00
8	0.00	0.21	0.00
9	0.10	0.00	0.00
10	0.00	0.00	0.00

Table: Cash Flows of the option under this (sub)-optimal strategy.
Discounting and taking the mean gives a value of 0.07798.
Exercising in $t = 0$ is not optimal.

Approximate dynamic programming

This idea can be extended to multiple stopping problems, giving good bounds for the value of a swing option for any stochastic process that can be simulated.

I don’t want to show the formulas with the technical details.

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Thank you for your attention!